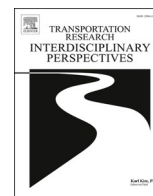


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Operating at the individual level: A review of literature and a research agenda to support needs-forward models of transport resource allocation

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ABSTRACT

Transportation equity is defined as the fair distribution of transportation system outcomes, costs, benefits, and services to individuals or communities. For transportation investments, plans, and projects to advance equitable outcomes, proactive equitable allocation mechanisms are necessary. Proactive equitable allocation mechanisms in transportation refer to the mathematical methods that distribute transportation system outcomes across individuals, particularly disadvantaged individuals, who are expected to benefit from the potential intervention. While existing conceptual and empirical equity literature establishes the need for proactive equity-driven transportation interventions, literature reviews on equity-focused mathematical approaches to allocate transportation system resources are limited. To this end, this literature review draws from the transportation engineering and operations research literature that focuses on the design of proactive or speculative mathematical methods through simulation and optimization to allocate transportation system outcomes in an equitable way. Through categorizing the literature based on the mathematical method first, followed by the application, we find a variety of conceptual and mathematical definitions of equity applied to an array of modes, applications, and scales of intervention. We also review the research that incorporates individual disadvantage status in determining the optimal allocation of transportation outcomes and find that many examples define broad categories of disadvantage across population groups, rather than mathematically model the behavior and needs of disadvantaged individuals. These findings are critical in laying out future research directions in equitable allocation methods that authentically center the positionalities of disadvantaged individuals, while also balancing other important transportation system objectives and public engagement strategies.

1. Introduction

Transportation equity is defined as the fair distribution of transportation system outcomes, costs, benefits, and services to individuals or communities (Litman, 2022). For transportation investments, plans, and projects to forward equitable outcomes, proactive equitable allocation mechanisms are necessary. Proactive equitable allocation mechanisms in transportation refer to the mathematical methods that distribute transportation system outcomes across individuals, particularly across disadvantaged individuals, receiving the outcomes of a potential intervention (Krapp et al., 2021; Lucas, 2012; Pereira et al., 2017; Ryerson et al., 2022; Wachs and Kumagai, 1973). While decades of research and practice directly engaging diverse transportation system users – especially those with financial and physical constraints – have showcased that user needs are highly varied across a population, engineering methods have long prioritized impedance-based (i.e., time, cost)

optimization in transportation. Without methods to know what projects may best deliver outcomes for diverse individuals, the expenditure of federal dollars – for example, through the 2021 \$1.2 trillion Bipartisan Infrastructure Law – is at risk of investing in projects that may unintentionally place disproportionate burdens on disadvantaged populations.

While existing conceptual and empirical equity literature establishes the need for *proactive* equity-driven transportation interventions (Karner and Niemeier, 2013; Pereira et al., 2017), literature reviews on equity-focused mathematical approaches to allocate transportation system outcomes are limited. Planning approaches in transportation equity often skew towards assessing and/or evaluating outcomes of transportation interventions across diverse populations, including but not limited to, accessibility outcomes, environmental justice, and support for disadvantaged groups. This set of literature is *ex-post based on revealed data* in understanding outcomes after the implementation of an

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intervention. While there is a growing subset of equity literature taking a natural experiment approach to measure the causal effects of transportation interventions, this literature is based on naturally occurring phenomenon rather than simulating an intervention (Lee et al., 2022). To this end, this literature review draws from the transportation engineering and operations research literature that focuses on the design of *proactive* or *speculative* mathematical methods through simulation and optimization to allocate transportation system outcomes in an equitable way. In this review, we focus on literature pertaining to the development of new transportation methods that yield equitable allocation outcomes, and we weave in the urban planning literature to provide context in shaping needs and possibilities.

We explore and present the extent to which the transportation engineering and operations research literature has developed mathematical models to allocate transportation system outcomes based on equity and fairness objectives. Through categorizing the literature based on the mathematical method first, followed by the application, we find a variety of conceptual and mathematical definitions of equity applied to an array of modes, applications, and scales of intervention. In doing so, we assess the strengths and limitations of each mathematical implementation of equity and provide application-specific guidance to practitioners in identifying projects that drive equity outcomes. We also review the research that incorporates individual disadvantage status in determining the optimal allocation of transportation outcomes and find that many examples define broad categories of disadvantage rather than mathematically model the behavior and needs of disadvantaged individuals. These findings are critical in laying out future research directions in equitable allocation methods that authentically center the positionalities of disadvantaged individuals, while also balancing other important transportation system objectives and public engagement strategies.

The remainder of the paper is structured as follows: we discuss the methodological framework for identifying and categorizing transportation equity methods (Section 2), summarize the literature selection procedure of this literature review (Section 3), synthesize equity methods (Sections 4 and 5), discuss applications of equitable allocation methods in transportation research (Section 6), explore the strengths, limitations, and applicability of equitable allocation methods (Section 7), and end with future research directions (Section 8) and a conclusion (Section 9).

2. Transportation equity framework

Guo et al. (2020) present a three-step equity assessment framework on which to categorize transportation equity studies: population, cost/benefit measurement, and equity method. Population measurement requires defining the population for whom we evaluate transportation system outcomes and comparing the outcomes across different subsets or groups within a population. The second step is cost/benefit measurement to quantify the impact of transportation system outcomes on a population. The third step is inequality measurement, or the equity method, where outcomes are judged to be equitable across populations.

The first step in categorizing transportation equity studies is to define the populations for whom transportation system outcomes are distributed. Population measurement requires dividing the population into subgroups or other units of analysis, and then comparing transportation system outcomes across these different groups (Guo et al., 2020). The exact method of grouping populations into subgroups is often intertwined with the third step, inequality measurement. As an example, populations can be defined as spatial units of analysis (e.g., census tract, traffic analysis zones), thus assuming homogeneity within a population. Additionally, populations can be defined at the individual or subgroup-level, differentiated based on priority levels, socioeconomic status, mobility needs, and/or health and environmental vulnerability (Litman, 2022). *This consideration of homogeneity and heterogeneity across populations is the basis of horizontal and vertical equity and is the primary focal*

point throughout this review.

The second step is cost/benefit measurement to quantify transportation outcomes that may impact a population as a result of an intervention. Here we introduce the concept of a metric, or transportation system outcome, that is used to guide the application of the method. In other words, the inputs or units that must be distributed across a population are the metrics. For example, the accessibility metric could be distributed such that disadvantaged travelers or those with historically low transportation access may benefit the most from a transportation system intervention (Ruiz et al., 2017; Santos et al., 2008; Wei et al., 2017). Other metrics include environmental emissions and safety (Guo et al., 2020), as well as disaster relief material and flight/airline slots (Karsu and Morton, 2015).

The third, and the focal point of our review, is inequality measurement, or the equity method. Equity methods refer to how transportation system outcomes or metrics are judged to be equitable by comparing outcomes across subgroups (Guo et al., 2020). There are two primary methods through which a metric can be distributed equitably: either as horizontal or vertical equity (Delbosc and Currie, 2011; Litman, 2022). Horizontal equity is defined as equalizing transportation system costs, benefits, and/or outcomes across populations. The key methodological underpinning behind parameterizing horizontal equity is in ensuring an equal spread of outcomes across populations; that is, requiring that users have equal quantities of transportation system outcomes regardless of the characteristics of the individuals impacted. The second method is vertical equity, which further defines population subgroups such that each subgroup may be determined based on heterogeneous socioeconomic status or need (Guo et al., 2020). Vertical equity then seeks to allocate transportation benefits to favor those groups, optimizing allocation of transportation benefits to serve prioritized or disadvantaged groups.

Throughout this review, we utilize Guo et al. (2020)'s equity assessment framework but focus critically on the third step, inequality measurement. In the discussion of equity methods in Sections 4 and 5, population definition and inequality measurement are often linked and thus we discuss them jointly for the bulk of this review after Section 3. In distinguishing between "metrics" and "methods," we separate the mathematical implementation of equity (methods) from the actual mechanism through which equity is implemented (metrics). We first discuss the equity methods corresponding to horizontal and vertical equity, followed by the application as the means to evaluate the efficacy of the method applied to different metrics and modes. *Thus, a contribution of this literature review is the separation of the method and application as we draw from diverse mathematical implementations of equitable transportation resource allocation.*

3. Literature selection

Literature selection focuses primarily on scholarship that has measured transportation equity outcomes across the applications of urban mobility, vehicle routing, and air transportation. Using keyword search combinations with the words "equity," "justice," "fairness," "transportation," "logistics," "priority," "aviation," and "accessibility" in Google Scholar, we employed the following criterion to determine the studies that were used in this literature review: First, the article should be relevant to the topic of equity and fairness in the broad fields of transportation engineering, logistics, or urban planning. Articles on spatial justice theory in transportation are excluded from our literature scope as the focus of this review is on mathematical methods to equitably allocate transportation system resources. Second, the article should be published in a peer-reviewed journal. Once these papers were identified, for select papers, we looked at backward citations to gather the paper's references, and forward citations, to gather the papers that reference the paper.

Fig. 1 summarizes the methodology of selecting literature and categorizing the papers we review. Throughout this review, we synthesize

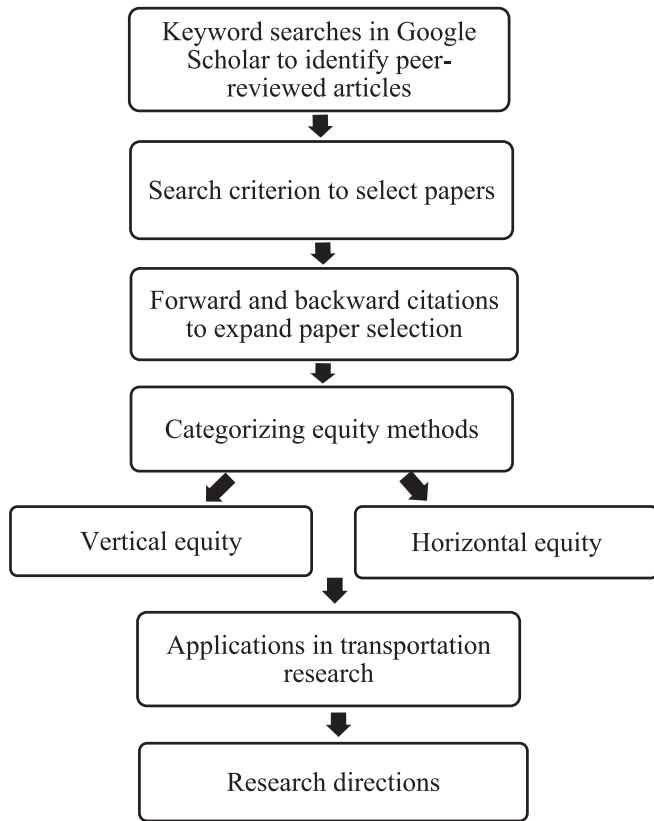


Fig. 1. Summary of literature selection methodology.

the literature across a variety of transportation modes and applications, inclusive of urban mobility and air transportation, as well as the broader vehicle routing scholarship pertaining to package delivery and disaster relief routing. Our literature selection includes (but is not limited to) public transit and roadway network optimization, flight slot allocation and scheduling, transit accessibility, and relief routing and delivery. Regardless of the metric or mode in question, each example provides a methodological implementation or consideration of horizontal and/or vertical equity. We thus categorize the papers based on first, the method (either horizontal or vertical equity), and then based on the application. In the next section, we discuss the framework for defining populations and mathematical methods pertaining to horizontal equity across our selected application areas.

4. Horizontal equity

The first approach to parameterize equity is horizontal equity, which is defined as ensuring an equal spread of outcomes across populations. In Section 4.1, we first define populations as applied to horizontal equity, and in Section 4.2, we discuss the mathematical methods to ensure horizontal equity outcomes.

4.1. Population definition

In horizontal equity, transportation system outcomes are distributed across populations considered as “equal in ability and in need” (Caggiani et al., 2017; Delbosc and Currie, 2011; Litman, 2022). Populations are thus divided homogeneously into subgroups or units regardless of their needs and abilities, such that the allocation of outcomes does not favor one particular group over another. To divide populations into subgroups, the exact unit of analysis over which equal allocations of resources can be made may vary depending on the application (Litman, 2022). For example, allocations can be made across spatial units (e.g.,

census tracts, traffic analysis zones), households, individual persons, demand nodes, or in the application of air transportation, individual flights or airlines. These units of analysis may vary spatially but are not explicitly differentiated with respect to the unit’s unique positionality and needs. Throughout this section and in our discussion of horizontal equity methods in Section 6, it is important to note that the unit of analysis may change depending on the problem context, varying from spatial units to individuals to demand nodes.

4.2. Horizontal equity methods

Based on homogenous definitions of population groups, horizontal equity methods then seek to ensure an equal allocation of transportation system outcomes across all units. The mathematical methods to parameterize horizontal equity are categorized in the following two main approaches: 1) distributional equity (Section 4.2.1) through a) inequality indices (Section 4.2.1.1) and b) minimizing the distance from equal allocation (Section 4.2.1.2), as well as 2) minimizing the distance of outcomes across units (Section 4.2.2).

4.2.1. Distributional equity

Distributional equity methods ensure an equal allocation by first quantifying the deviation in outcomes across a population and minimizing or constraining this deviation in an optimization problem. Quantifying and minimizing the deviation across outcomes requires first defining an equal allocation scheme, such that whatever outcome is being allocated, all users receive equal amounts. This baseline equal allocation can be determined by inequality indices or as a user-prescribed scheme, which is the focus of this section.

4.2.1.1. Inequality indices. Most used in the economics and operations research literature, inequality indices measure the spread of a metric across individuals, groups, and/or spatial units. Inequality indices assign values across a population such that $F(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ is a mapping of a distribution of outcomes ($x \in \mathbb{R}^n$) to a scalar value in \mathbb{R} (Karsu and Morton, 2015). Inequality indices also have the property that they take a value of 0 to represent perfect equality; that is, each unit receives the same outcome. In equitable resource allocation, these indices can be used as part of an objective function to either be minimized or used in a constraint to restrict the value of the inequality index beyond a pre-specified threshold. The two inequality indices we discuss in this section are the Gini coefficient and Theil index, which are the most commonly used metrics. To illustrate these methods, we create a standard set of notation and define the variables below. We define an arbitrary metric X that can represent any transportation system outcome and calculate each index across a set of units N , which in total is represented by the value n .

N : set of all units within a population (ex. spatial units, individuals, or demand nodes), $j \in N$

X_j : scalar value of the metric X corresponding to unit j

\bar{X} : average of the metric X across all units $j \in N$ in the population.

n : total number of units that correspond to the population. N

Y_k : proportion of unit k relative to the entire population.

4.2.1.1.1. Gini coefficient. The Gini coefficient is the most used inequality index, often used to measure income inequality (Karsu and Morton, 2015). Examples that use the Gini coefficient define equity as the level of fairness of the distribution of impacts, which acknowledges the differences in outcomes across populations (Feng et al., 2010; Kaplan et al., 2014; Santos et al., 2008). Below, we present two definitions of the Gini coefficient.

Definition 1. The Gini coefficient is based on the Lorenz curve, which is a graph of population proportion on the horizontal axis and the income share on the vertical axis. The index is defined as taking the difference in population proportion $(Y_k - Y_{k-1})$ weighted by X_k , which is the value of the metric for a unit k .

$$Gini_1 = 1 - \sum_{k=1}^N (Y_k - Y_{k-1})(X_k + X_{k-1}) \quad (1)$$

Definition 2. In an alternate formula that is mathematically equivalent to Definition 1, the Gini coefficient can also be defined as half of the relative mean absolute difference. This method sums the differences in the metric value X_k across all the units in the set N , weighted by the average metric value and the total number of units. In this definition, the Gini coefficient measures the difference between the deviation in the metric X across units.

$$Gini_2 = \frac{\sum_{j \in N} \sum_{k \in N} |X_j - X_k|}{2n^2 \bar{X}} \quad (2)$$

4.2.1.1.2. Theil index. The Theil index is used to quantify the degree of inequality as it accounts for minimizing inequalities between units, thus ensuring an equal spread of outcomes (Santos et al., 2008).

Definition 3. The Theil index sums the differences in the distribution of the metric weighed by the total population n .

$$Theil = \frac{1}{n} \sum_{k=1}^N \frac{X_k}{\bar{X}} \ln \frac{X_k}{\bar{X}} \quad (3)$$

4.2.1.2. Minimizing deviation from equal allocation. In a second approach to ensure distributional equity and minimize the deviation from equal allocation, an objective function can directly quantify the spread of outcomes and penalize this deviation. This interpretation of equity requires first, defining the baseline level of equality such that whatever metric is being allocated, all users are given equal amounts. Equity is then measured by minimizing the displacement from the equal allocation of outcomes; implicitly, this assumes that while an equal allocation cannot be obtained, there must be an equitable way to take away resources, outcomes, or metrics. In Section 6, we discuss examples, largely skewing towards flight slot allocation, that minimize the deviation from the equal allocation.

4.2.2. Minimizing distance of outcomes across units

The crux of horizontal equity relies on the equal distribution of outcomes across units in a system where each unit is parameterized homogeneously across a population. To ensure that all units have equal outcomes (where units can refer to spatial units, individuals, households, or demand nodes), optimization methods will minimize the distance of outcomes across units in the objective function, resulting in optimal solutions where all units receive equal or close to equal outcomes. Mathematically, this can be achieved in two ways: first, through majorization, which defines a preordering on the decision vector, ensuring that the components of one vector are more evenly distributed than the components of another vector (Ball et al., 2009). Additionally, the objective function can explicitly minimize the difference in outcomes across individuals and/or groups to ensure that individuals and/or groups receive similar outcomes.

5. Vertical equity

The second major approach to parameterizing equity is vertical equity; recall that vertical equity is defined as allocating outcomes to favor certain individuals more than others, dependent on differentiated need and/or status. In Section 5.1, we first outline methods to define population subgroups for vertical equity, and in Section 5.2, we discuss the methods to favor disadvantaged individuals in equitable allocation.

5.1. Population definition

In vertical equity, the critical first step is in determining population subgroups by differentiating units within a population according to

heterogeneous status or need. Similar to horizontal equity, the unit of analysis over which resource allocations are made can vary depending on the application and can range from spatial units to individuals to demand nodes. To uniquely determine these population subgroups, disadvantage status is often used with vertical equity, which refers to the unit's social positionality, mobility needs, health and environmental vulnerability, or propensity to take and/or need transportation access (Karner and Niemeier, 2013; Litman, 2022; Wachs and Kumagai, 1973). Transportation disadvantage can stem from an individual and/or group's inability to experience transportation system benefits due to cost, safety, or mobility burdens, or it can refer to transportation services failing to support needs (Aivinho and Zuidgeest, 2019; Lucas, 2012).

As a starting point for vertical equity, there are several dimensions to categorize individuals into levels of priority and/or need that range from geographic to financial to time-based exclusion (Lucas, 2012). For example, low-income individuals and/or groups with greater transit dependence are more prone to unstable transportation access induced by limited private vehicle access and income (Grengs, 2015; Lowe and Mosby, 2016; Manaugh and El-Geneidy, 2012; Morris et al., 1979; Ryerson et al., 2022; Wachs and Kumagai, 1973). Other factors that contribute to disadvantage status in transportation planning are environmental justice (Karner et al., 2020; Rowangould et al., 2016) and gender, for example, low-income women who face a unique set of household, childcare, and financial constraints (Blumenberg, 2004). Because of these mobility and socioeconomic constraints, disadvantaged populations are more likely to be disconnected from economic and social activities and opportunities (Grengs, 2015). Towards targeting interventions towards communities of need, Section 5.2 summarizes the vertical equity methods to favor disadvantaged populations in transportation resource allocation.

5.2. Vertical equity methods

Based on the identification of priority groups and/or individuals, the next step in vertical equity is to optimize for prioritized groups such that transportation system outcomes directly favor these individuals. To do so, we draw from the transportation engineering and operations research literature to provide examples of studies that both identify priority groups and develop methods to optimally allocate outcomes to serve priority groups.

5.2.1. Proportionality

Proportionality as an equity method has been discussed in the urban planning and operations research fields from both philosophical and mathematical perspectives. The basis of proportionality ensures that outcomes are distributed in proportion to the share that the group represents relative to the entire population (Bills and Walker, 2017; Martens et al., 2012). This is a step beyond distributing outcomes equally regardless of individual circumstances (horizontal equity), rather, distributing outcomes relative to the population share considers the subgroup's positionality with respect to the broader population. To optimally distribute resources to ensure proportionality, objective functions consist of the following simplified functional form (Eq. (4)) with the variables consistent with the notation presented in Section 4.2.1.1. Eq. (4) denotes that the sum of outcomes X_i across all individuals i in subgroup j should be proportional to the size of subgroup j , ensuring that outcomes or resources are distributed according to the size of the subgroup. Eq. (4) can be alternatively interpreted as minimizing the average of the outcome X_i across all subgroups j (Jacquillat and Vaze, 2018; Zhang and Waller, 2019).

J : set of all subgroups within a population, $j \in J$

N_j : set of individuals corresponding to subgroup j

n_j : size of the subgroup $j \in J$

X_i : scalar value of the outcome X corresponding to individual i

$$P = \min \frac{1}{n_j} \sum_{i \in N_j} X_i \forall j \in J \quad (4)$$

5.2.2. Rawlsian max-min approach

The Rawlsian max-min approach is a classic method in the transportation and operations research fields to incorporate equity and fairness in resource allocation (Bertsimas et al., 2011; Karsu and Morton, 2015). Rawls' justice philosophy is the theoretical underpinning behind the max-min method by distributing outcomes to favor those units, individuals, and/or demand nodes that are most disadvantaged, thus seeking to improve the material standing of disadvantaged units. We categorize the Rawlsian max-min approach as a vertical equity method because the method implicitly defines populations with respect to heterogeneous status and/or need, even though it does not explicitly differentiate units based on disadvantage status. The single objective max-min method works to optimize the outcome of the worst-off unit either by maximizing (minimizing) the minimum (maximum) outcome directly in the objective function or through a constraint that ensures that the worst-off unit receives a pre-specified minimum outcome level.

Expanding on the single objective max-min approach, the lexicographic max-min methods is also commonly used to ensure fairness in allocation schemes. The lexicographic max-min method works as follows: it first optimizes for the unit receiving the worst outcome, and then for the unit receiving the second and third worst outcomes, and so on, in order to achieve the most equitable allocation scheme for the individuals that are not the worst-off (Lehuédé et al., 2020; Luss, 1999; Matl et al., 2018; Vossen and Ball, 2006). An important theoretical property of solutions from max-min approaches is that they are Pareto optimal, such that increasing an outcome for an individual (i.e., making them better off) results in decreasing an outcome for another individual (i.e., making them worse off) (Bertsimas et al., 2011).

5.2.3. Elevating discretized groups in allocation

Other vertical equity allocation problems have further defined populations through discretized groups across a population, differentiated based on heterogeneous characteristics, and have then allocated resources to most benefit these disadvantaged groups. These characteristics are commonly based on levels of priority (e.g., the urgency of the type of resource to be allocated, the need associated with the individual to be served) or other disadvantage status (e.g., low-income, low access). The circumstances of these groups are then incorporated into the objective function to minimize the sum of unsatisfied demand, service, and/or need, weighted by the cost of not satisfying that group's demand. This ensures that groups with the highest priority (i.e., the groups with the largest cost of unsatisfied demand) may be served first. Additionally, mathematical constraints can also ensure preferred treatment for the most disadvantaged groups such that a minimum level of demand is satisfied for these groups. Finally, priority groups can be determined and elevated based on differentiated travel time sensitivities or values of time, thus refining the categorical binary notion of high vs. low priority. Similarly, objective functions may incorporate these values of time to ensure that groups with the highest time sensitivity may be favored in resource allocation.

5.2.4. Priority scoring functions

In Section 5.2.3, priority groups were largely determined based on the level of urgency or severity of need for the subgroup, with several methods considering priority as individuals' differentiated values of time. While aggregating individual needs to a group level may ease computational issues, individual-level differences that are crucial in equity-driven planning may be masked (Bills and Walker, 2017; Carleton and Porter, 2018). To further refine the definition of prioritized groups and incorporate individuals' needs into equitable allocation, some works of scholarship have considered mapping priority as a scalar quantity through a priority function. This priority function inputs a

vector of variables and outputs a priority score; the magnitude of which determines the unit's level of priority (Gutjahr and Fischer, 2018; Jin et al., 2015; Rivera-Royero et al., 2016; Zhu et al., 2019). Based on these definitions of priority scores, objective functions can then allocate outcomes to favor individuals with the most priority or penalize weighted unmet demand such that optimal resource distributions favor results where the most disadvantaged individuals are served first.

6. Equitable allocation methods in transportation research

Based on the mathematical definitions of horizontal and vertical equity described in Sections 4 and 5, we now review the literature in transportation research that implements equitable allocation methods across three core application areas: 1) urban mobility, 2) vehicle routing, and 3) air transportation. For each application area, we categorize each example as either horizontal or vertical equity and describe the mathematical implementation of the equity method. Table 1 summarizes these methods for urban mobility applications, and Table 2 summarizes the methods for vehicle routing and air transportation.

6.1. Urban mobility

Equitable allocation in urban mobility centers around transit service intervention (the optimal allocation of transit services, largely focusing on transit frequency setting problems) and network design problems for public transit or private vehicle usage (road network design scenarios are optimized according to time, cost, and equity objectives). Both applications evaluate or distribute the impacts (e.g., accessibility, cost, time) of transit service or network redesign interventions across populations through horizontal and vertical equity methods.

6.1.1. Transit service intervention

Public transit provides affordable access to a variety of essential social and economic opportunities (e.g., employment, health care). While there are varied definitions of accessibility in the transportation planning scholarship, accessibility has traditionally referred to the ease at which individuals access opportunities through the transportation system (Kaplan et al., 2014). A range of literature – both qualitative and quantitative – suggests that certain groups face barriers in transit access

Table 1
Summary of equity methods in urban mobility applications.

	Method	Transit service intervention	Transportation network design
Horizontal equity	Inequality indices	Delbosc and Currie (2011), Kaplan et al. (2014), Ruiz et al. (2017), Zuo et al. (2020)	Bao et al. (2022), Caggiani et al. (2017), Feng et al. (2010), Feng and Zhang (2014), Santos et al. (2008), Sumalee et al. (2009), Zhang and Waller (2019)
	Minimizing distance of outcomes across units	Ferguson et al. (2012)	Caggiani et al. (2017), Camporeale et al. (2016), Chen and Yang (2004), Fan and Machedehl (2011), Meng and Yang (2002), Wang and Chen (2021)
Vertical equity	Proportionality		Zhang and Waller (2019)
	Rawlsian max-min approach Elevating discretized groups	Ruiz et al. (2017), Wei et al. (2017)	Miyagawa (2009) Santos et al. (2008), Hai and Xiaoning (2002)

Table 2
Summary of equity methods in vehicle routing and air transportation.

	Method	Vehicle routing	Air transportation
Horizontal equity	Distributional equity		Guo et al. (2022)
	Minimizing distance from equal allocation	Huang et al. (2012), Vitoriano et al. (2011)	Balakrishnan and Chandran (2010), Barnhart et al. (2012), Bertsimas and Gupta (2016), C. Chin et al. (2021), Glover and Ball (2013), Jones and Lovell (2014), Kim and Hansen (2013), Kotnyek and Richetta (2006), Kuhn (2013), Mukherjee and Hansen (2007), Samà et al. (2017), Vossen et al. (2003), Vossen and Ball (2006)
	Minimizing distance of outcomes across units	Lin et al. (2011)	Ball et al. (2009)
Vertical equity	Proportionality		Guo et al. (2022), Jacquillat and Vaze (2018), Manley and Sherry (2010), Zografos and Jiang (2019)
	Rawlsian max-min approach	Campbell et al. (2008), Ibarra-Rojas and Silva-Soto (2021), Ransikarbum and Mason (2016)	Balakrishnan and Chandran (2010), Samà et al. (2017)
	Lexicographic max-min fairness	Lehuédé et al. (2020)	Jacquillat and Vaze (2018), Vossen and Ball (2006)
	Elevating discretized groups	Afshar and Haghani (2012), Balci et al. (2008), Chiu and Zheng (2007), Lin et al. (2011), Salmerón and Apte (2010), Sheu (2014), Tofighi et al. (2016), Yi and Ozdamar (2007)	Ball et al. (2020), Vlachou and Lovell (2013)
	Continuous priority scores	Gutjahr and Fischer (2018), Jin et al. (2015), Rivera-Royero et al. (2016), Zhu et al. (2019)	

related to vehicle ownership, income, gender, sexuality, and race (Aivinheno and Zuidgeest, 2019; Bills and Walker, 2017; Blumenberg, 2004; Geurs and van Wee, 2004; Wachs and Kumagai, 1973). Equity studies often measure the distribution of transit accessibility or service levels across these disadvantaged groups (Geurs and van Wee, 2004; Guo et al., 2020), while several examples seek to determine optimal transit interventions or operational decisions (e.g., transit headways) by maximizing transit accessibility for disadvantaged travelers. However, few examples truly focus on allocating transit services based on individual-level needs, which presents a key research gap in equitable allocation for transit service provision.

Evaluating disparities in transit accessibility across levels of disadvantage is critical to ensure that transit services are being allocated in order to serve the populations that have historically suffered from low transit access (Carleton and Porter, 2018). This area of equity studies results in quantitatively describing the state of inequities of transit

service across groups, rather than proactively determining interventions that maximize equity. To this end, the Gini coefficient and Theil index are commonly used methods to quantify the distribution or spread of transit accessibility across socioeconomic groups (Kaplan et al., 2014; Zuo et al., 2020). Additionally, Lorenz curves (Definition 1) can provide a visual representation of the level of equality of transit supply across an entire population, or across subgroups divided based on socioeconomic status (Carleton and Porter, 2018; Delbosc and Currie, 2011). The results illustrate that certain subgroups of the population may experience disproportionate levels of transit service, thus highlighting the need for targeted efforts to address inequities (Carleton and Porter, 2018; Delbosc and Currie, 2011).

While these examples express the need for targeted transit interventions towards individuals suffering from low accessibility, other examples determine the optimal allocation of transit services to most benefit disadvantaged individuals. Ruiz et al. (2017) use the Gini coefficient (Definition 2) to find optimal bus frequencies over socially excluded populations and identify areas that may have high demand but less service. Wei et al. (2017) implement a bi-objective optimization problem to simultaneously optimize for equitable access (defined by summing the total disadvantaged individuals served by transit) and efficiency. Additionally, to expand access via transit for low-income populations to employment opportunities, Ferguson et al. (2012) determine the optimal bus frequencies by minimizing the difference in transit and private vehicle access. However, these results do not consider how disadvantaged individuals' priorities or demand patterns may impact the optimal allocation of transit services and instead focus on ensuring transit coverage for disadvantaged individuals. This is further discussed in Section 8 as directions for future research.

6.1.2. Transportation network design

Traditional network design problems focus on minimizing cost or travel time objectives from the transit agency perspective, rather than considering potential reduced benefits towards disadvantaged populations and the undue burdens they face (Ferguson et al., 2012; Jahn et al., 2005; Santos et al., 2008). To address these concerns, several examples incorporate equity objectives to design transportation networks for private vehicle or public transit users by considering the network distributions of travel time or cost reductions based on horizontal equity (Bao et al., 2022; Caggiani et al., 2017; Camporeale et al., 2016; Chen and Yang, 2004; Fan and Machemehl, 2011; Meng and Yang, 2002), and additionally under road charging schemes (Sumalee et al., 2009; Hai and Xiaoning, 2002). This seeks to ensure a maximal level of improvement across outcomes before and after the proposed network design intervention. For example, Meng and Yang (2002) and Fan and Machemehl (2011) incorporate a parameter in their bi-level optimization problem that equalizes the benefits of the network redesign across users. Caggiani et al. (2017), Camporeale et al. (2016), and Chen and Yang (2004) undertake a similar approach but consider "fuzzy" optimization to account for uncertainty in their constraints and objective function. Zhang and Waller (2019) develop link-based equity metrics using proportionality (Eq. (4)) as the average dispersion of excess travel time and energy impacts for network design expansion.

Several approaches optimize for horizontal equity by distributing the impacts (e.g., time, cost, accessibility) of network redesign scenarios equally across populations. Feng and Zhang (2014) and Feng et al. (2010) use bi-level optimization to trade off equity and cost, where they maximize equity using inequality indices. Other approaches use inequality indices to evaluate the level of fairness of each possible network design scenario, either for airport transit networks (Definition 1; Bao et al., 2022) or for private vehicle and transit networks across congestion pricing scenarios (Definition 3; Camporeale et al., 2019). Wang and Chen (2021) allocate active transportation investments by maximizing total accessibility and minimizing the difference in accessibility between low-access and high-access neighborhoods to ensure that all populations receive similar outcomes. Miyagawa (2009) also

optimizes for horizontal equity in evaluating the hierarchy of road networks in Tokyo by calculating the “ratio of road areas that minimizes the maximum travel time.”

While these examples acknowledge the disproportionate impacts of transportation system outcomes on certain populations, they still do not explicitly favor disadvantaged individuals in resource allocation. To this extent, [Hai and Xiaoning \(2002\)](#) divide the population into subgroups based on their values of time in a discrete multiclass network equilibrium model and bi-level optimization approach of network redesign with consideration of tolls. In considering socioeconomic-based need, [Santos et al. \(2008\)](#) maximize three different equity measures under network improvements: accessibility to low-accessibility centers, the Gini coefficient, and the Theil index, finding that the optimal road network design differs based on the equity measure being maximized.

6.2. Vehicle routing

Vehicle routing is a type of allocation problem that distributes relief material or other goods to populations under constrained vehicle supply and transportation costs. Traditional vehicle routing problems minimize total network transportation costs while ensuring adequate demand satisfaction levels. These studies often minimize the total travel distance of vehicles in allocating relief material to communities of need rather than accounting for how some individuals may have to wait longer for relief even though system-level distance is minimized ([Karsu and Morton, 2015](#)). To this end, there is a robust branch of scholarship in disaster relief routing that considers the differentiated positionalities of vulnerable individuals through vertical equity methods. In seeking to incorporate equity into vehicle routing problems, and additionally balance equity objectives with efficiency, many approaches skew towards the Rawlsian max-min fairness approach ([Campbell et al., 2008](#); [Lehuédé et al., 2020](#)).

The horizontal equity interpretation in disaster relief routing ensures that all nodes or individuals receive a similar level of relief, regardless of their level of priority or disadvantage status. Equity is parametrized as the maximum deviation proportional to the demand, which is then set to be upper bounded as a constraint by a pre-specified value ([Vitoriano et al., 2011](#)). Maximizing a minimum fraction of fulfilled demand ([Ibarra-Rojas and Silva-Soto, 2021](#); [Ransikarbum and Mason, 2016](#)), minimizing the differences in demand satisfaction rates across nodes ([Lin et al., 2011](#)), or penalizing unmet demand ([Huang et al., 2012](#)) ensures that the distribution of relief material is spread out evenly across demand nodes.

However, these approaches do not capture the differentiated levels of need across individuals, wherein certain individuals may require urgent need, and relief material may be limited. The equity question thus focuses on optimally allocating relief material such that individuals with the most priority or disadvantage receive the care they need. Several examples distinguish between critical (those in need of emergency evacuation) and non-critical (those who can wait to be evacuated) demand nodes and then minimize the weighted sum of unsatisfied demand ([Afshar and Haghani, 2012](#); [Balcik et al., 2008](#); [Lin et al., 2011](#); [Tofighi et al., 2016](#); [Yi and Ozdamar, 2007](#)) or use bi-objective optimization to first optimize for the critical group and then the non-critical group ([Salmerón and Apte, 2010](#)). Beyond the binary critical vs. non-critical differentiation, there are other approaches to define discretized priority groups based on levels of need (e.g., special need, emergency) or based on demographics (e.g., elderly, women with young children). Based on these distinctions, the optimal allocation of relief material will fulfill the demand of each group and then maximize a weighted sum of demand such that individuals with the highest priority receive relief material first ([Chiu and Zheng, 2007](#); [Sheu, 2014](#)).

Beyond discretized priority groups, individual priority and need can be quantified using continuous functions that map a set of individual characteristics to a discrete value representing the level of need of the individual. Priority score functions can take as inputs: the waiting time

for relief, characteristics of the demand nodes, and product relevance, and output a value, which is then weighed in the objective function to minimize the weighted sum of unsatisfied demand ([Rivera-Royero et al., 2016](#)). Based on individuals' severities of injury through survival probabilities, [Jin et al. \(2015\)](#) maximizes the number of survivors in a disaster weighted by their survival probability, which is dependent on the victim's injury. An alternative interpretation of need in disaster relief is the deprivation cost function which quantifies individuals' suffering if they did not have relief services. [Zhu et al. \(2019\)](#) assign an absolute deprivation cost to high and low priority groups (which is modeled in three states as a function of time that includes exponential and linear valuation of relief) and minimize the absolute and relative deprivation costs. Combining deprivation costs with the Gini coefficient ([Definition 2](#)) to enforce more equitable solutions, [Gutjahr and Fischer \(2018\)](#) find that minimizing deprivation costs alone may disproportionately negatively impact the minority population.

6.3. Air transportation

Applications of equitable allocation in air transportation focus on fairly distributing limited capacity units and/or delays across flights and/or airlines. In capacity allocation with flights, airlines have a pre-existing schedule that ensures scheduled departure times for flights. When airport capacity is reduced, airlines must reduce their schedule; how the schedule must be drawn down and reallocated for each airline is the core of the equity question. Across air traffic management and airline scheduling applications, we find that there is no standard conceptual or mathematical definition of equitable slot allocation; rather, allocation methods will assign arrival and departure slots in capacity-constrained airports according to several metrics ([Guo et al., 2022](#)). The most equitable distribution is the adherence to flights' claims to their original slots, or the ration-by-schedule scheme ([Balakrishnan and Chandran, 2010](#); [Ball et al., 2009](#); [Bertsimas and Gupta, 2016](#); [Glover and Ball, 2013](#); [Jones and Lovell, 2014](#); [Kotnyek and Richetta, 2006](#); [Kuhn, 2013](#); [Mukherjee and Hansen, 2007](#); [Samà et al., 2017](#); [Vossen et al., 2003](#); [Vossen and Ball, 2006](#)). Second, slots can be allocated to flights based on the deviation from air traffic delays ([Kim and Hansen, 2013](#)), and third, slots can be allocated based on an airline's share relative to the total slots requested ([Zografos and Jiang, 2019](#)).

The ration-by-schedule (or alternatively, first scheduled, first served) scheme preserves airlines' original schedules of slots and is classically considered the most equitable allocation scheme to distribute slots to airlines under capacity-constrained situations ([Guo et al., 2022](#); [Kim and Hansen, 2013](#); [Mukherjee and Hansen, 2007](#)). Scholars have interpreted fairness in slot assignment by minimizing the deviation from the ration-by-schedule scheme ([Balakrishnan and Chandran, 2010](#); [Glover and Ball, 2013](#); [Kotnyek and Richetta, 2006](#); [Kuhn, 2013](#); [Samà et al., 2017](#); [Vossen et al., 2003](#)). To do so, [Vossen and Ball \(2006\)](#) ensure adherence to the ration-by-schedule scheme through a lexicographic max-min criterion. Additionally, to avoid extreme deviations from the optimal allocation scheme, convex disutility functions can penalize large deviations between the actual and ration-by-schedule allocation ([Mukherjee and Hansen, 2007](#)), the controlled schedule ([Barnhart et al., 2012](#); [Chin et al., 2021](#)), or the minimal delay scenario as the optimal allocation ([Jones and Lovell, 2014](#)). Besides convex disutility functions, minimizing the maximum delay over all flights will prevent extreme inequities and unfair bias towards certain flights ([Balakrishnan and Chandran, 2010](#); [Samà et al., 2017](#)). In the context of ground delay program management in assigning slots to flights, [Ball et al. \(2009\)](#) consider equity as the equal spread of delay across flights using majorization, a method that reduces the distance of delays associated with individual flights.

Equitable allocation in air transportation also encompasses inter-airline equity to ensure an equitable distribution of schedule displacements or delays across airlines. [Manley and Sherry \(2010\)](#) develop an airline equity metric to ensure that airlines with more flights receive

more delay, while Guo et al. (2022) use the Gini coefficient to distribute delays across airlines. Jacquillat and Vaze (2018) optimize airline schedules based on both on-time performance and interairline equity. Their objective function lexicographically minimizes airline disutilities of displacing a flight, where the displacement is proportional to the airline's number of flights (Eq. (4)). Similarly, Zografos and Jiang (2019) use max-min fairness to ensure that the "worst case of un-fairness differs as little as possible from the average fairness," where fairness is the airline's total displacement proportional to the number of slots that the airline has requested.

Other vertical equity perspectives in ensuring equitable slot allocation, although limited, can also account for the positionalities and preferences of individual airlines. To protect smaller airlines, restricting the market power of large airlines by allocating more slots to smaller airlines can account for the unique positionality of small or new entrant airlines (Ball et al., 2020). The notion of priority has also been parameterized through the usage of airline preferences, requiring airlines to assign a priority number to specific flights (on a scale from 1 to 4) in an airspace flow program, which is then used in flight slot allocation mechanisms (Vlachou and Lovell, 2013).

7. Synthesis of equitable allocation methods in transportation

We now summarize the strengths and limitations of the reviewed equitable allocation methods (Section 7.1), followed by a discussion of the applicability of these methods to certain applications, modalities, and scales of intervention intended to guide applications for both transportation researchers and practitioners (Section 7.2).

7.1. Strengths and limitations of equitable allocation methods

Sections 4 through 6 have discussed equitable methods in transportation based on horizontal and vertical equity and their applications in urban mobility, vehicle routing, and air transportation. The varied approaches to parameterize equity highlight the strengths of certain methods; however, these implementations can be conceptually and mathematically limited due to several factors. While horizontal equity methods may be computationally feasible, the main challenge is that horizontal equity optimizes for the spread of outcomes (e.g., transit accessibility, travel time, demand satisfaction) equally across all subgroups regardless of individual or group-level characteristics. Vertical equity methods address this shortcoming yet are both limited and highly varied in how populations are defined based on disadvantage status.

Horizontal equity: Horizontal equity has the potential to ensure that all individuals and/or groups in a population have equal access to transportation system outcomes. For example, it is important to ensure that all members of a population have public transit access and can benefit equally from transit service improvements. However, the major limitation of horizontal equity is that it focuses on equalizing impacts across populations, rather than correcting historical inequities that have led to unfavorable outcomes for certain individuals and/or groups. In certain applications, minimizing the deviation from an equal allocation or minimizing the distance in outcomes across units may result in disproportionately negatively impacting certain units over others as it does not account for the unique positionalities of subgroups across a population. For example, adhering to the ration-by-schedule scheme, which preserves airlines' original schedules of slots, may not necessarily elevate small or new entrant airlines by restricting the market power of large airlines. In vehicle routing, equally distributing relief material can neglect the demand nodes and/or individuals with the most urgent need.

Distributional equity through inequality indices is classified as horizontal equity yet can be used to highlight how transportation system outcomes fall disproportionately across a population (Carleton and Porter, 2018; Delbosc and Currie, 2011). Thus, inequality indices have the potential to be incorporated under a vertical equity paradigm; moreover, they are favorable in that they are explicit measures that can

be easily integrated into decision-making processes. However, the specific definition of fairness through an inequality index relies on the theoretical and mathematical properties of each index (e.g., Gini coefficient, Theil index) which may lead to simplifying complex equity issues. Thus, using inequality indices often results in a tradeoff between tractability and satisfying equity concerns (Karsu and Morton, 2015).

Vertical equity: While some vertical equity methods are straightforward in their implementation and thus are widely accepted measures to capture equity, they can still be conceptually limited. For example, the Rawlsian max-min approach abounds in transportation equity research due to ease of implementation (Karsu and Morton, 2015). Yet the challenge of implementing max-min fairness arises from its assumption that all individuals or users in the system should have an equal right to resources. Thus, max-min fairness does not consider that different individuals may want to receive more or less share than others due to their circumstances. While the lexicographic max-min approach addresses the shortcomings of the max-min approach to optimize for other individuals (beyond just those in the worst-off position), the method does not allow for elevating groups and/or individuals who may have increased need; rather, it assumes that every entity should have equal access to resources.

While vertical equity methods ensure that disadvantaged individuals and/or groups are favored in resource allocation, the examples reviewed are highly varied in how populations are defined based on disadvantage status. Populations can be defined based on the size of the group relative to the size of the population, as discretized priority groups, or based on a set of priority indicators. In doing so, each method of defining populations can introduce new biases: for example, proportionality can be biased because it skews towards subgroups that have a high pre-existing share relative to the general population. Thus, proportionality masks those subgroups that may have smaller shares or sizes relative to the entire population but have higher disadvantage status (Bills and Walker, 2017). While proportionality considers subgroup-level status, it does not necessarily favor the groups with the most need (such as small or new entrant airlines) which may be critical in certain applications (as discussed in Section 7.2).

7.2. Applicability of equitable allocation methods

Across the different applications in Section 6, there is not one equitable allocation method suitable to all transportation applications. Each method carries distinct theoretical and practical interpretations, and its applicability depends on the scope and considerations of the problem at hand. However, based on the engineer or planner's attitude towards equity and the extent to which equity should be considered, certain methods may be relevant to specific applications.

The two general classes of methods, horizontal and vertical equity, may be well-suited to certain applications in transportation based on the user's interpretation of equity. Horizontal equity applies to problems where positive impacts (e.g., transit accessibility) or negative impacts (e.g., transportation costs or travel time burdens) of an intervention need not disproportionately affect one group. For example, all populations should have equal access to transit services to enable social and economic mobility. In this case, distributional equity through inequality indices is favorable for many applications in assessing dispersion in transportation system outcomes and obtaining understandings of existing degrees of inequity. To identify potential interventions, minimizing the deviation from an equal allocation of outcomes and minimizing differences across outcomes is applicable to limit gaps in the distribution of outcomes, for example, to ensure that transit interventions do not disproportionately benefit certain groups over others.

Vertical equity methods apply to applications in which favoring disadvantaged groups in the allocation of transportation system outcomes is the objective. The Rawlsian max-min and lexicographic max-min approaches are suitable if the application is focused on improving the outcomes of the worst-off entity, for example, to prevent extreme

delays with flights and/or airlines or travel time burdens for particular individuals in a network. Beyond just optimizing for the worst-off unit, applications in disaster relief routing have incorporated levels of urgency and priority, and vertical equity is explicitly used to ensure that individuals receive relief material according to their level of need. Vertical equity methods can be used in the particular case for distributing resources (e.g., slots) to small or new entrant airlines such that they receive preferable outcomes compared to larger airlines with more market power. In the application of transit service interventions, vertical equity may be used to identify operational decisions that most benefit disadvantaged individuals.

While both horizontal and vertical equity methods are generalizable to different scales of interventions as seen in the numerous examples reviewed, computational tractability can be an issue for large-scale vertical equity problems and thus may limit applicability. Vertical equity-based optimization problems require parameterizing the unique circumstances of subgroups within a population and allocating outcomes to favor the most disadvantaged groups, which may intensify the computational cost in solving these problems. Additional planning objectives (e.g., efficiency, operating cost), while necessary in developing implementable solutions for transportation practice, may conflict with vertical equity objectives and may pose additional computational issues. Vertical equity methods may thus be better suited for smaller-scale optimization problems when disaggregated, individual-level data is available.

8. Research directions

Section 7 provides the foundation for cross-application research directions (Section 8.1), in which we outline four focus areas for future transportation equity research. In Section 8.2, we discuss application-specific research directions across all of the three transportation domains discussed in Section 6.

8.1. Cross-application research directions

Cross-application research areas are motivated by the methodological gaps in the transportation literature, as well as the objectives under the Biden-Harris Administration's 2021 Justice 40 Initiative, which guides the allocation of federal funding towards transportation equity by ensuring that at least 40% of the benefits from the U.S. Department of Transportation (USDOT)'s transportation initiatives go towards disadvantaged communities. In this section, we contextualize our research directions with the Justice 40 Initiative objectives to highlight how our proposed focus areas align with existing approaches to address equity.

Focus 1: Enrich the unit of analysis in defining populations by collecting individual-level data on diverse traveler experiences. Examples in this literature review utilize aggregated units of analysis, often as spatial units (e.g., census tracts), groups of individuals, or demand nodes, thus presenting challenges in identifying interventions that will most address the nuanced needs of diverse individuals. This is driven by the lack of individual-level data in transportation planning, resulting in masking individual details that are critical in determining sources of disadvantage (Bills and Walker, 2017). Individual-level data collection is key to understanding user perspectives before implementing an intervention in order to best support the individuals who will directly benefit from transportation enhancements (Brown, 2022). For future work in defining population needs, data collection should incorporate the components of disadvantage from the Justice 40 Initiative, such as geography (e.g., rural, suburban, urban), community (e.g., tribal), and demographics (e.g., seniors, women, youth). Through disaggregated data collection, methods of determining individual need can be more reflective of individual circumstances in order to better identify targeted interventions towards addressing the needs of these individuals.

Focus 2: Incorporate disadvantaged traveler decision-making and behavior in equitable allocation methods. A broad area of improvement

across existing transportation research and practice methods is the consideration of the complex decision-making processes of disadvantaged individuals. For example, under the Justice 40 Initiative, the USDOT's Equitable Transportation Community (ETC) Explorer provides users with an understanding of how a community is experiencing transportation disadvantage to help ensure that the benefits of investments are addressing disadvantage. However, the ETC Explorer is focused on measuring the level of disadvantage of a geographic area, rather than estimating the behavior and demand patterns of disadvantaged travelers for a potential project (El Zarwi et al., 2017). Future methodological development around vertical equity methods must first incorporate more nuanced definitions of disadvantage (Focus 1) and then model the complex decision making of these disadvantaged travelers before allocating transportation system outcomes. These methods should be calibrated with qualitative survey data to fully identify the diverse sets of constraints across disadvantaged individuals, aligned with Focus 1.

Focus 3: Balance multiple transportation system objectives in identifying equity-maximizing interventions. Many of the studies reviewed acknowledge that optimizing solely for vertical equity may conflict with other planning objectives (e.g., efficiency, agency operating costs, environmental impact). For instance, equity (as defined by accessibility) and transit efficiency (as defined by travel time or operational costs) can have conflicting objectives, as allocating transit services to provide increased accessibility for the individual traveler may incur additional agency costs. While many examples consider the tradeoffs between equity and efficiency, other critically important outcomes are left out, such as safety and environmental outcomes (Haight, 1994). Future methodological research in equity-focused transportation must weigh multiple outcomes (e.g., safety, environment) with vertical equity objectives. While incorporating equity considerations will further aid in determining optimal transportation interventions, it is critical to also address the computational limitations in solving multi-objective optimization problems.

Focus 4: Evaluate and iterate on equitable allocation methods through public engagement and data collection. Upon implementing an intervention that seeks to deliver equity outcomes, community engagement is required to evaluate how interventions are achieving their intended objectives. A key aspect of the Justice 40 Initiative ensures that community leaders have been engaged throughout a project's development and execution. This community input will ensure that projects that are equitably allocating transportation system outcomes are benefitting those in need (Brown, 2022; Karner and Marcantonio, 2018). Engineers, together with planning agencies and city authorities, should seek public input to evaluate and monitor identified transportation system interventions. Engineers and planners can continuously incorporate public feedback into new iterations of equitable allocation methods to avoid the risk of implementing transportation interventions that may benefit the individuals with the least need or have unanticipated consequences on disadvantaged individuals.

8.2. Application-specific research directions

This final section summarizes application-specific research directions to provide guidance to researchers and practitioners in identifying potential projects that drive equity outcomes in the urban mobility, vehicle routing, and air transportation fields.

While there is abundant consideration of disadvantaged individuals in transit service provision, much of the literature skews toward the evaluation of planning interventions. Using existing definitions of quantifying transit inequities, future work may allow local transit agencies and metropolitan planning agencies to make operational decisions (e.g., new routes, headways) based on maximizing equity. Moreover, the few examples that have used equity to determine optimal transit interventions have not considered how disadvantaged individuals' priorities or demand patterns may impact the allocation of

transit services. Rather, as these examples have focused on ensuring transit coverage for disadvantaged individuals, future work should incorporate individuals' needs and priorities in identifying potential interventions. Similarly, network design problems consider spatial equity with accessibility outcomes; however, they do not incorporate the decision-making behavior of disadvantaged individuals. This lack of a person-based approach may be due to limitations on data availability concerning individuals' circumstances and mobility patterns. Future research must enhance data collection to understand the needs and priorities of disadvantaged individuals and incorporate them into equitable allocation methods.

As discussed in Section 6.3, equity in air transportation skews towards adhering to the first scheduled, first served scheme as the standard of equity in flight slot allocation. Additionally, interairline equity ensures that flight delays and schedule displacements are allocated proportionally based on the airline's share of slots or number of flights. Future directions of research should incorporate vertical equity perspectives around elevating smaller and new entrant airlines by specifically incorporating their status as compared to larger airlines. A small scope of literature has focused on aviation accessibility and equity in measuring access to intercity destinations via air transportation (Gosling, 2000; Karam et al., 2022). Yet the examples that model aviation accessibility and equity are not focused on proactively determining optimal aviation system interventions to best promote access to intercity destinations. In the field of air transportation, and broader intercity transportation (inclusive of rail transportation), future research directions should develop equitable allocation methods for disadvantaged individuals who may need increased access to air, rail, or other intercity modes.

9. Conclusion

This review has synthesized equitable allocation methods in the transportation engineering, planning, and operations research fields spanning a variety of modes, applications, and scales of interventions. Equity methods are divided into horizontal equity, where outcomes are allocated across a homogenous population, and vertical equity, where outcomes are allocated to favor certain individuals within a population of heterogeneous needs. We first provided an overview of Guo et al.'s (2020) transportation equity assessment framework (i.e., population, metrics, and methods) and then synthesized the most commonly used methods in horizontal and vertical equity, presenting each method's mathematical implementation and further extensions. We then reviewed the applications of equitable allocation methods across three different transportation domains: urban mobility, vehicle routing, and air transportation. Based on the examples reviewed, we discussed the strengths, limitations, and applicability of each equity method to specific modes, applications, and scales of intervention. Our recommendations for future research directions build upon this discussion to guide methodological and practical transportation equity research and identify research directions for diverse applications.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

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